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PRS

THE INCORPORATION OF RETIREMENT LOSS ESTIMATES IN THE PERSONNEL PLANNING PROCESS

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Office of Personnel

United States Department of Agriculture

Washington, D. C.

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SUMMARY AND HIGHLIGHTS

Retirement losses represent one considerable component involved in estimating future personnel needs in an ongoing organization and in planning replenishment. Reported here are some techniques for collecting and analyzing information to estimate future retirement losses in a more systematic fashion than is commonly applied.

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The Incorporation of Retirement Loss Estimates in the Personnel Planning Process

The logistics of personnel planning require that we have available useful information on the numbers and kinds of people that we will need in the future to operate our organizations. This information is most useful when it is available sufficiently in advance of the specific need to enable the organization to do an effective job of obtaining and of developing the right kinds of people. Otherwise, we are at the mercy of the supply and demand that chance provides at the moment that our requirements for personnel become immediate.

The recognition of the value of long range planning of this sort is reflected in the recent issuance (November, 1964) of the U. S. Civil Service Commission entitled "Federal Workforce Outlook: Fiscal Years 1965-1968."

Retirement losses represent one considerable component involved in estimating future personnel needs in an ongoing organization and in planning replenishment. Reported here are some techniques for collecting and analyzing information to estimate future retirement losses in a more systematic fashion than is commonly applied.

The Concept of a Retirement Index (RI)

We would like to derive, for each individual, an index of probability of that individual's retiring within a given number of years. We can do this by using certain items of information that are available on the people in the workforce. These items of information should be readily available and change systematically or remain constant during the coming years. Thus, for example, with the passage of a year an employee becomes one year older and acquires one year more towards his retirement eligibility.

Collecting and Analyzing the Data

As a first step in finding items of information that are pertinent to retirement prediction, the monthly reports of retirement submitted to the Office of Personnel by the agencies were reviewed. It was found that, with the exception of one agency, two categories of information on recent retirees were available: Age (or date of birth) and number of years of service (or service computation date). Accordingly, a

Retirement Index was formed for each retiree by adding together his age and years of service at the time of retirement (RI = Age + Years of Service) and an analysis of these data was performed on those 2,465 persons who were optional or mandatory General Service (GS) retirees between January 1961 and March 1964[1]. (The scoring protocol for these data is given in Appendix A).

Comparison of Agency and Annual Differences on the Retirement Index

This analysis sought to answer two questions:

(1) Are there agency differences and

(2) Are there year-to-year differences on the RI?

To answer these questions, a multiple regression framework was used. To denote membership in a given agency, a retiree was given a score of one if he retired from that agency; zero if he retired from some other agency. Repeating this scoring procedure for each agency gives us as many variables as there are agencies. Similarly, if a person retired during a particular calendar year, he was scored one for that year; zero if he retired during some other year. A hypothetical data matrix is given below for seven retirees, three agencies and three years:

			Agency			Year		
		RI	ARS	ASCS	AMS	1961	1962	1963
Retiree	1	76	1	0	0	1	0	0
	2	81	1	0	0	0	0	1
	3	78	0	1	0	0	1	0
	4	97	0	1	0	1	0	0
	5	89	0	1	0	0	1	0
	6	99	0	0	1	0	1	0
	7	102	0	0	1	0	0	1

[1] Retirements due to refusal to relocate were omitted from the analysis where these were indicated on the reports. Only two persons in this group of retirees were 71 or older and had 16 or more years of service.

Reading across, we see that the first retiree has an RI of 76, retired from ARS (1 entry in the ARS column) and retired in calendar year 1961 (1 entry in the 1961 column). The agency membership variables are mutually exclusive as are the year of retirement variables (viz. a retiree has membership in only one agency and retires during only one calendar year).

The first question we would like to answer is: "Is the squared multiple correlation (SMC) with RI appreciably increased when both agency membership and year of retirement variables are used, as compared to when only agency membership variables are used?" If there is an appreciable increase in the SMC it would mean that there are differences on the RI that can be attributed to both agency membership and year of retirement. If there is not an appreciable increase in the SMC, this means that we do not have to take into account the year in which retirement occurred.

Exhibit 1 compares the SMC obtained when only agency membership variables are taken into account with the SMC obtained when both agency and annual (or year-to-year) variables are taken into account[2].

Exhibit 1

Comparison of SMC's When Annual Differences Are Taken into Account

Agency and Annual Variables	SMC = .30
Agency Variables Alone	SMC = .30
Difference	.00

The difference between the SMC's in Exhibit 1 is zero and hence we can disregard mean differences between years.

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[2]Agency by year interactions were not deemed to be of sufficient interest to be included in this analysis. An explanation of the manner in which multiple regression analysis utilizes information is given in Appendix B. For a more detailed explanation of this approach the reader is referred to Bottenberg, R. A. & Ward, J. H. Jr., "Applied Multiple Linear Regression," Technical Documentary Report PRL-TDR-63-6, March, 1963, 6570th Personnel Research Laboratory, Aerospace Medical Division, Air Force Systems Command, Lackland Air Force Base, Texas. Appendix C presents the means, standard deviations and intercorrelations from this analysis.

The next question that arises is: "How much of a difference exists between the agencies on the Retirement Index?" This question can be answered by observing the successive increase in the SMC as agency variables are brought into the regression analysis. The computer program for the regression analysis is written so that the variable which will provide the next greatest increment in the SMC is brought in at each stage. The results of this analysis are presented in Exhibit 2.

Exhibit 2

Increase in SMC as Agency Differences are Taken into Account

Order of Inclusion	Magnitude of SMC
ARS	.07
ASCS	.09
AMS	.09
FS	.09
SCS	.09
Other	.09
FHA	.09

Exhibit 2 shows that there is no increase in the SMC after the agency membership variables for ARS and ASCS are entered into the regression analysis. This means that only ARS and ASCS differ appreciably from the other agencies on the average RI's. These differences are more evident in Exhibit 3 which gives agency averages for the RI as well as for age and years of service.

Exhibit 3

Average Retirement Index, Age and Years of Service for Each Agency

Agency	RI	Age	Years of Service	Number of Retirees
ARS	96.28	64.87	31.41	682
ASCS	87.81	64.89	22.92	362
AMS	92.24	65.64	26.60	319
FS	91.71	62.34	29.37	459
SCS	91.15	64.78	26.37	389
Other	90.68	64.98	25.70	116
FHA	90.57	65.53	25.04	138
Department Total	92.27	64.53	27.74	2465

Exhibit 3 shows that ARS has the highest average RI and ASCS has the lowest. These differences are due primarily to the ARS retirees having the greatest number of years of service and ASCS retirees the fewest number of years of service. The greater number of years of service of ARS retirees might reflect the fact that many of these people were recruited directly from college into USDA jobs that existed prior to the great upsurge of USDA employment that occurred in the late 1930's. Hence these retirees, as a group, have more service than the others. The shorter service of ASCS retirees might be accounted for by the fact that when the ASCS programs were started during that period, many persons in their 30's and 40's were recruited, and hence they tend to retire with fewer years of service.

It is also of interest to note that the FS has the youngest age at retirement. Thus all the other agencies tend to retire at about 65 whereas FS retires at about 62.

The department overall, retires at about age 65 with 28 years of service. Retirees are more similar with respect to age than they are with respect to years of service[3].

Comparison of Agency Differences on Their Retirement Distributions

One of the desirable products of a study such as this is a retirement probability table. Such a table enables one to make a statement about the likelihood of an individual with a particular RI retiring in, say, a five-to-ten year period. It is also desirable to have one table as widely applicable as possible. This latter condition can be met to the extent that the agency retirement distributions are similar.

Exhibit 4 compares these retirement probability distributions. They were obtained by starting at the low end of the RI scale and computing the index value for the 10th, 25th, 50th, 75th and 90th percentiles respectively.

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[3]The standard deviations for the Department overall are: Age = 4.17, Years of Service = 8.94, RI = 9.23.

Exhibit 4

Comparison of Agency Retirement Index Values
for Selected Percentiles

Agency	Percentiles 10	25	50	75	90
ARS	83	89	95	102	109
ASCS	76	82	87	92	97
AMS	78	84	89	98	107
FS	80	86	91	96	101
SCS	81	86	90	95	99
Other	79	84	90	94	102
FHA	81	85	91	95	98
Department Total	80	85	91	97	104

Exhibit 4 is read as follows: In ARS, 10% of the retirees have a RI value of 83 or less; 25% have an RI value of 89 or less; 50% an RI value of 95 or less; 75% an RI value of 102 or less and; 90% have an RI value of 109 or less.

Inspection of Exhibit 4 shows that the agency distributions tend to be consistent with the exception of ARS and ASCS. Again, ARS tends to have higher index values at each percentile while ASCS is lower. If a constant of 5 were subtracted from each ARS index value and a constant of 5 were added to each ASCS index value, these differences would tend to be corrected. These corrections can be incorporated in the retirement probability table.

Development of a Retirement Probability Table

A graphic illustration of the relationship between the Retirement Index and the Probability of Retirement is given in Exhibit 5.

The retirement probability table in Exhibit 6 was developed by starting at the low end of the RI scale and computing the cumulative percent of retirees for selected scale values. These percentages can then be interpreted as probabilities.

Exhibit 6 can be read by entering the first column with one's RI value which is determined by adding age in years and length of service. The numerical values in the appropriate row indicate the chances in 100 of retiring in that number of years. Thus let us take as a hypothetical case a person who has an RI value of 66. There is 1 chance in 100 that he will retire in the coming year, 4 chances in 100 that he will retire within 3 years, 9 chances in 100 for 5 years, etc.

EXHIBIT 5

RELATIONSHIP BETWEEN RETIREMENT INDEX AND PROBABILITY OF RETIREMENT

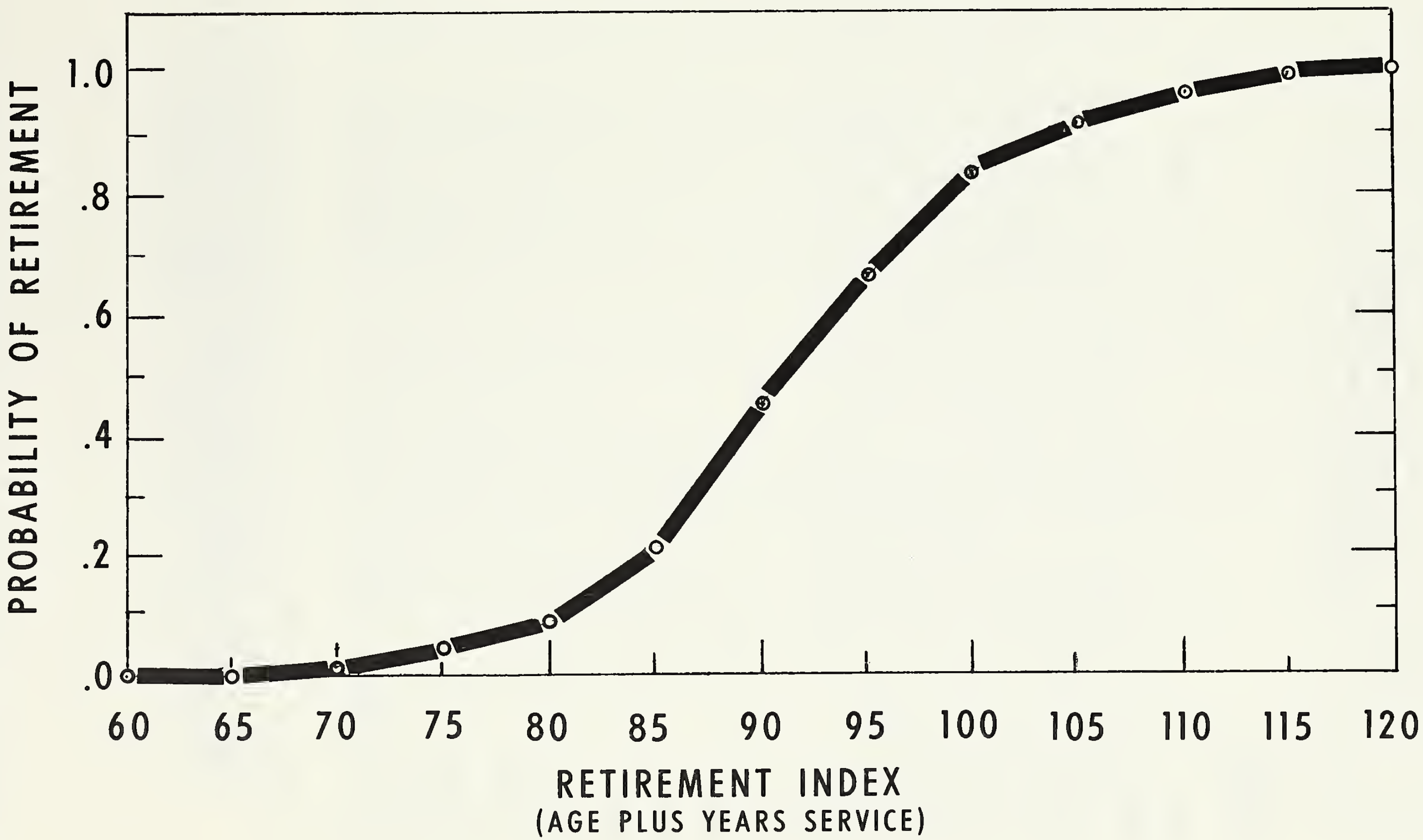

Exhibit 6

Retirement Probability Table
Probability of Retirement in ___ Years

Current RI Value*	1	3	5	8	10	15	20	25	30
21-25	0	0	0	0	0	0	0	4	21
26-30	0	0	0	0	0	0	1	9	45
31-35	0	0	0	0	0	0	4	21	66
36-40	0	0	0	0	0	1	9	45	83
41-45	0	0	0	0	0	4	21	66	91
46-50	0	0	0	0	1	9	45	83	96
51-55	0	0	0	1	4	21	66	91	99
56-60	0	0	1	4	9	45	83	96	100
61-65	0	1	4	9	21	66	91	99	100
66-70	1	4	9	21	45	83	96	100	100
71-75	4	9	21	45	66	91	99	100	100
76-80	9	21	45	66	83	96	100	100	100
81-85	21	45	66	83	91	99	100	100	100
86-90	45	66	83	91	96	100	100	100	100
91-95	66	83	91	96	99	100	100	100	100
96-100	83	91	96	99	100	100	100	100	100
101-105	91	96	99	100	100	100	100	100	100
106-110	96	99	100	100	100	100	100	100	100
111-115	99	100	100	100	100	100	100	100	100
116 or Greater	100	100	100	100	100	100	100	100	100

*ARS subtract 5 from index value before reading into the table.
ASCS add 5 to index value before reading into the table.
Current RI value equals <u>age</u> plus <u>years</u> <u>of</u> <u>service</u>.

These values for future years are obtained by projecting the current retirement probabilities. Thus, for every year in the future, one's RI increases by a value of 2 (1 for age and 1 for a year of service). These projections assume that the retirement probabilities will remain roughly the same. An annual or biennial check can be made to ascertain to what extent changes have occurred. If changes have occurred, new rates can be easily computed.

Applications of the Retirement Index

In order to demonstrate one possible application of the Retirement Index some data available from another study were analyzed. Data on age, years of service and GS level were obtained on current employees by randomly sampling administrative units in ARS. This sampling procedure yielded a total of 3,069 employees. A breakdown by GS level and RI values was obtained [4] and expected losses for 3, 5 and 10 years were computed. This table and the computational procedures involved are given in Appendix D. Expected losses for a given GS level are computed by multiplying the number of employees with a specified RI value by the proportion of expected losses for that RI value from Exhibit 6. These products are then summed to get the total loss for that GS level. An example of this procedure is given in the following table.

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[4] After elimination of certain obvious conversion errors (e.g. employees having more years service than their age would allow) the correlation between age and years of service was .68 for 2,727 employees.

Corrected Retirement Index Values (ARS)	GS-15	Probability of Retirement in Three Years	Product
41-45	3	.00	.00
46-50	4	.00	.00
51-55	5	.00	.00
56-60	2	.00	.00
61-65	7	.01	.07
66-70	1	.04	.04
71-75	6	.09	.54
76-80	0	.21	.00
81-85	1	.45	.45
86-90	1	.66	.66
91-95	1	.83	.83
SUM	31	SUM	2.59

The projection is that about 3 out of the 31 GS-15's will retire within the next 3 years.

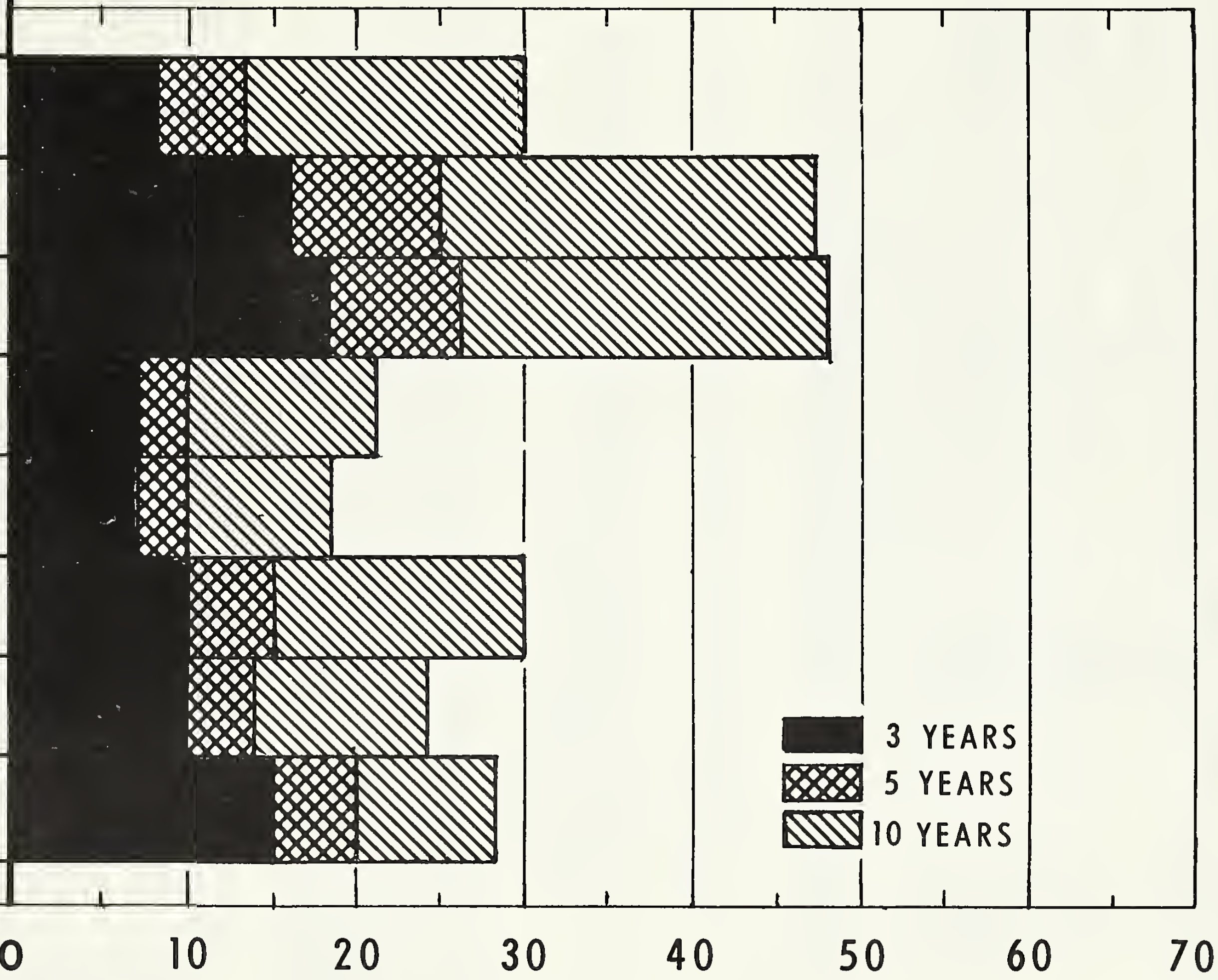
The results of this analysis for a 3, 5 and 10 year period are presented graphically in Exhibit 7. This graph shows that approximately: (1) 13% of the GS-15's, 25% of the GS-14's, 26% of the GS-13's, 10% of the GS-12's and 10% of the GS-11's will retire during the next 5 years; (2) 30% of the GS-15's, 47% of the GS-14's, 48% of the GS-13's, 21% of the GS-12's and 18% of the GS-11's will retire during the next 10 years.

These results can be integrated with certain assumptions about the effects of program changes on the numbers of employees. Thus, if one assumes that the program or programs involving these employees will remain relatively stable and losses will be replenished from the next lower GS level, then the expected losses for each level indicate the number of promotions that will be made. Also, the total losses will indicate the number of new hires that will be needed at the entry level. An example is listed below.

EXHIBIT 7

PERCENTAGE LOSSES EXPECTED IN 3, 5 AND 10 YEARS

GS LEVELS	CURRENT NUMBER
15	31
14	126
13	102
12	356
11	379
8-10	482
5-7	1,097
1-4	496



GS Level	Current Number	Expected Number Loss for Next 3 Years	Promotion Likelihood
15	31	3	-
14	126	20	2:100
13	102	18	22:100
12	356	25	12:100
11	379	27	17:100
8-10	482	48	19:100
5-7	1097	111	13:100
1-4	496	76	-

In this example let us assume that replenishments are made from the next lower GS level for GS-5's and above. This means that there will be:

(1) 3 GS-14's promoted to GS-15 to replace those retiring GS-15's,

(2) 20 GS-13's promoted to GS-14 to replace those retiring GS-14's, plus 3 GS-13's promoted to GS-14 to replace those promoted to GS-15.

Thus, a total of 23 GS-13's will have to be promoted to replenish losses due to retirement and promotion. One can see that the losses are pushed down to the next lower level so that the number needed at the 5 to 7 level will be the sum of all the losses for GS-5 and above (viz. 3+20+18+25+27+48+111 = 252). Thus, a total of 252 entry level employees will be needed during this period.

The column labeled "Promotion Likelihood" gives the chances in 100 of a person at a given grade level being promoted during the next 3 years. These figures were obtained by dividing the upward replenishments for that grade level by the current number of people in that grade level. Thus, the chances of promotion for a GS-14 are very slight but there are 22 chances in 100 that a GS-13 will be promoted during this period. Similar calculations could be made for a 5 and 10 year period. The implications for planning, training, advancement, and the like are quite clear.

These calculations were made under the assumption that the program would remain stable. Anticipated program expansion or contraction would alter the number at different levels and hence increase or decrease the replenishments correspondingly.

Although these examples are grossly simplified, when the computer aspect of the Management of Human Resources Program (MOHR) becomes operational, it will be a simple matter to perform more refined and complex analyses and accumulate historical data for specific occupational groups. Retirement loss estimates can be integrated with projected losses for other kinds of attrition such as voluntary quits, changes to other series, etc., so that comprehensive manpower projections can be made. Indeed, with modern computer techniques it is possible to simulate the personnel flow of an entire agency and USDA as a whole (see List of References).

List of References

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Appendix A

Scoring Protocol

Appendix A

Scoring Protocol

The agency membership variables were scored so that a 1 indicates that the person was a member of that agency upon retirement and an 0 indicates that he was a member of some other agency. They are mutually exclusive viz. memberships do not overlap.

1. Forest Service (FS) 1 or 0
2. Soil Conservation Service (SCS) 1 or 0
3. Agricultural Marketing Service (AMS) 1 or 0
4. Agricultural Research Service (ARS) 1 or 0
5. Agricultural Stabilization & Conservation Service (ASCS) 1 or 0
6. Farmers Home Administration (FHA) 1 or 0
7. Other[5] 1 or 0

The years of retirement were scored so that a 1 indicates that the person retired during that year and an 0 indicates that he retired during another year. These variables are also mutually exclusive.

8. 1961 1 or 0
9. 1962 1 or 0
10. 1963 1 or 0
11. 1964 1 or 0
12. Age in Years
13. Years of Service
14. Sum of 12 and 13.

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[5]Office of Management Services (OMS) was excluded from the analysis since their reports do not contain information on age at retirement. Reports from agencies that are now serviced by OMS were also excluded for the time period prior to the formation of OMS. The total number of retirees omitted because of these considerations was 161. Their average years of service at retirement was 30.62 and the average age (reported for only 59 retirees) was 63.33. The standard deviations were 8.48 and 4.49 respectively.

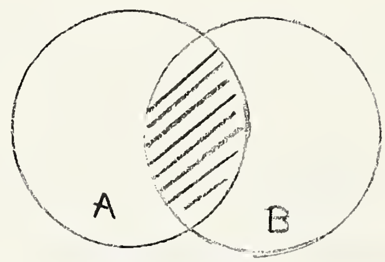
Appendix B

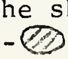
The Utilization of Information in Multiple Regression Analysis

The Utilization of Information in Multiple Regression Analysis

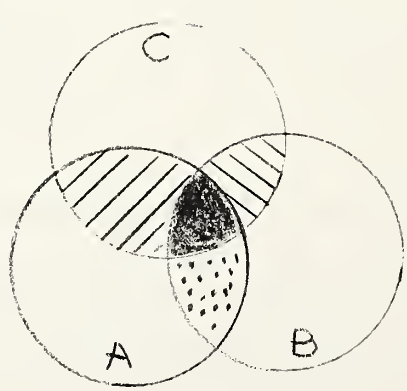
The manner in which this mathematical technique utilizes information is best illustrated by a few simple diagrams.

Suppose that we have two categories of information that are of interest to us. These can be represented by means of two overlapping circles or sets labeled A and B. The area of overlap, the shaded area, represents that information which is common to both A and B.



Suppose we wanted to add up everything in A and B by adding first everything in A and then everything in B. In doing this we would have counted everything in the shaded area twice, once when counting A and once when counting B. In order to be correct in our arithmetic we only want to count the shaded area once, hence we would subtract it once (A + B = A + B -).

Now let us complicate the picture a little by introducing a third set C,



and let us define the following areas:

- that information in the intersection of all three sets;

- that information common to B and C which A does not have;

- that information common to A and C which B does not have;

- that information common to A and B which C does not have;

If we wanted to add up everything in all three sets, A + B + C, we would end up counting the dotted and slanted areas twice each and the solid area three times. To correct for this we would subtract the dotted and slanted areas once each and the solid area twice, i.e.

$$A + B + C - \quad - \quad - \quad - 2 \quad .$$

We could make the picture increasingly complex by adding more sets of information; however, this example is sufficient. Let us ascribe some meaning to these categories of information.

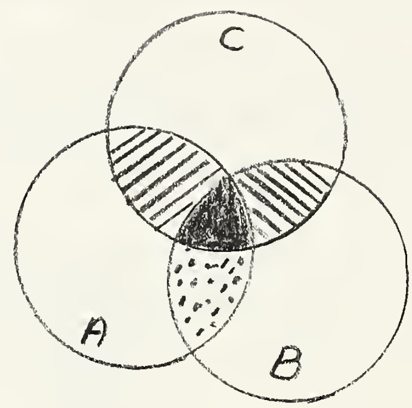
Let us now assume that A, B, and C are three measures on a group of people:

A - Score for each person representing his qualifications;

B - Score for each person representing his characteristics;

C - Score for each person indicating his "Readiness for Promotion."

In this context multiple regression analysis would attempt to ascertain the manner in which categories A and B are combined to arrive at an individual's "Readiness for Promotion" score. The outcome would be a set of weights assigned to each category of information so that when combined they provide the "Readiness for Promotion" score. The weights are assigned to these categories so as to maximize their relationship with the "Readiness for Promotion" score and minimize the amount of overlap between categories A and B. Thus, in the following example:



The solid area would be counted as part of A and then in assigning a weight to B the solid area would not be counted again. Thus, B would play a much smaller part in determining C than would A.

An individual's A score would then be multiplied by the weight for A, his B score by the weight for B, and then these two weighted scores would be summed in order to arrive at an estimated C score. A measure of the adequacy with which the C scores are estimated by this weighting system is called the multiple correlation. This value varies from 1.00 for perfect estimation to .00 for lack of any relationship.

Appendix C

Means, Standard Deviations and Intercorrelations for Agency and Annual Differences and for Age, Year of Service and Their Sum

Appendix C

Exhibit 8

Means, Standard Deviations and Intercorrelations for Agency and Annual Differences and for Age, Year of Service and Their Sum

		1	2	3	4	5	6	7	8	9	10	11	12	13	14	Mean	S.D.
1.	FS	1.00	-.21	-.18	-.30	-.20	-.12	-.11	.00	-.01	.02	-.02	.09	-.25	-.03	.19	.39
2.	SCS		1.00	-.17	-.27	-.18	-.10	-.10	-.04	.00	-.02	.09	-.07	.03	-.05	.16	.36
3.	AMS			1.00	-.24	-.16	-.09	-.09	.03	-.03	-.02	.05	-.05	.10	.00	.13	.34
4.	ARS				1.00	-.26	-.15	-.14	.01	.04	-.02	-.06	.25	.05	.27	.28	.45
5.	ASCS					1.00	-.10	-.09	-.01	-.01	.04	-.03	-.22	.04	-.20	.15	.35
6.	FHA						1.00	-.05	.00	-.01	.01	.00	-.07	.06	-.04	.05	.23
7.	Other							1.00	.01	.02	-.01	-.04	-.05	.02	-.04	.05	.21
8.	1961								1.00	-.44	-.44	-.17			.01	.30	.46
9.	1962									1.00	-.47	-.18			.00	.32	.47
10.	1963										1.00	-.18			.01	.32	.47
11.	1964											1.00			.00	.06	.25
12.	Age in Years												1.00	-.16		27.74	8.94
13.	Years of Service													1.00		64.53	4.17
14.	Sum of 12 and 13														1.00	92.24	9.23

N = 2465

Appendix D

Computational Rationale

The following computational rationale is outlined in terms of matrix algebra.

Let P be an s row by t column matrix where the columns indicate the different years for which projections are to be obtained and the rows indicate probability values for different Retirement Index (RI) values. Exhibit 6 is an example of such a matrix.

$$P\ (s,t) = \begin{bmatrix} p_{11} & p_{12} & \cdots\cdots\cdots & p_{1t} \\ p_{21} & p_{22} & \cdots\cdots\cdots & p_{2t} \\ \vdots & \vdots & & \vdots \\ \vdots & \vdots & & \vdots \\ p_{s1} & p_{s2} & & p_{st} \end{bmatrix}$$

Let G be an s row by u column matrix where the columns indicate different grade levels and the rows indicate different RI values.

$$G\ (s,u) = \begin{bmatrix} g_{11} & g_{12} & \cdots\cdots\cdots & g_{1u} \\ g_{21} & g_{22} & \cdots\cdots\cdots & g_{2u} \\ \vdots & \vdots & & \vdots \\ \vdots & \vdots & & \vdots \\ g_{s1} & g_{s2} & & g_{su} \end{bmatrix}$$

An example of a G matrix is given on the following pages. Let G_t indicate the transpose of G obtained by interchanging rows and columns.

Expected loss is computed by the matrix product:

$$(1)\ G_t\ P = L\ (u,t) = \begin{bmatrix} l_{11} & l_{12} & \cdots\cdots\cdots & l_{1t} \\ l_{21} & l_{22} & \cdots\cdots\cdots & l_{2t} \\ \vdots & \vdots & & \vdots \\ \vdots & \vdots & & \vdots \\ l_{u1} & l_{u2} & \cdots\cdots\cdots & l_{ut} \end{bmatrix}$$

Each column of the L matrix represents the losses for a given number of years hence and the rows indicate the GS level. Thus, l_{11} indicates the expected loss for the first GS level during the first time period e.g. GS-15 for 3 years hence.

Let 1 be an s rowed column vector whose elements are all unity. Then the numbers of employees at different grade levels are computed by:

$$(2) \quad G^t\, 1 = C\ (u,1) = \begin{bmatrix} c11 \\ c12 \\ \cdot \\ \cdot \\ cm,1 \end{bmatrix}$$

The numbers of employees remaining after the losses have occurred are computed by:

$$(3) \quad C1^t - L - R\ (u,t) = \begin{bmatrix} r11 & r12 & .. & r1t \\ r21 & r22 & .. & r2t \\ \cdot & \cdot & & \cdot \\ ru1 & ru2 & .. & rut \end{bmatrix}$$

Let M be a u rowed column vector where each row indicates replenishment from the next lower GS level. An element of unity indicates the occurrence of replenishment and zero for non-occurrence. Thus,

$$M\ (u,1) = \begin{bmatrix} m1 \\ m2 \\ m3 \\ \cdot \\ \cdot \\ \cdot \\ mu \end{bmatrix}$$

if m1 = 1 replenishment occurs from the next lower GS level, if m1 = 0 replenishment does not occur e.g. promotion from GS-14 to 15.

Under the assumption of replenishment from below (or promotion from within) the total number of replenishments needed at the entry level is given by:

(4) $M^t R = N(1,t) = [n1, n2, \ldots\ldots nt]$

But since $R = C1^t - L$, $C = G^t 1$, and $L = G^t P$ substituting these in (4) we find:

$$M^t (C1^t - L) = M^t (G^t 11^t - G^t P) = M^t G^t (11^t - P)$$

and letting $(11^t - P) = Q$

$$M^t G^t Q = N$$

Let F be a u by t matrix where the columns indicate the years and the rows indicate the anticipated numbers of employees at different GS levels. These anticipated numbers would be derived from a consideration of program changes.

$$F(u,t) = \begin{bmatrix} f11 & f12 & \ldots\ldots & f1^t \\ f21 & f22 & \ldots\ldots & f2^t \\ . & . & \ldots\ldots & . \\ fu^1 & fu^2 & \ldots\ldots & fu^t \end{bmatrix}$$

Thus f11 represents the number of employees anticipated in the first time period at the first GS level e.g. GS-15's three years hence.

Given an F matrix and an R matrix from equation (3) the difference between F and R will indicate the losses or gains that must occur to meet the program changes. Denote this matrix as D:

(5) $F(u,t) - R(u,t) = D(u,t)$

If the status quo is to be maintained this means that the anticipated numbers at different grade levels in some future time period will be much the same as they are now, hence:

$$F = C1^t$$

and substituting in (5):

$$C1^t - R = D$$

$$C1^t - C1^t + L = D \therefore L = D$$

Retirement Index Values	Corrected RI Values	ARS Grade Levels 1-4	5-7	8-10	11	12	13	14	15	Total
20 or less	15 or less	4	1	0	0	0	0	0	0	5
21-25	16-20	33	9	1	0	0	0	0	0	43
26-30	21-25	18	26	19	11	2	0	0	0	76
31-35	26-30	38	59	27	37	13	0	1	0	175
36-40	31-35	25	97	37	39	34	0	0	0	232
41-45	36-40	42	115	36	44	35	0	1	0	273
46-50	41-45	47	94	31	45	45	0	8	3	273
51-55	46-50	39	92	36	41	49	4	7	4	272
56-60	51-55	38	114	26	22	34	5	9	5	253
61-65	56-60	32	87	38	41	34	18	11	2	263
66-70	61-65	26	108	48	21	19	16	16	7	261
71-75	66-70	19	70	57	22	22	16	18	1	225
76-80	71-75	17	56	38	13	27	11	11	6	179
81-85	76-80	23	39	30	10	16	8	20	0	146
86-90	81-85	31	31	25	9	7	13	14	1	131
91-95	86-90	19	41	19	9	8	5	5	1	107
96-100	91-95	12	25	9	9	8	2	4	1	70
101-105	96-100	8	9	3	5	1	3	0	0	29
106-110	101-105	13	7	2	0	0	1	0	0	23
111-115	106-110	5	9	0	1	1	0	1	0	17
116+	111-115	7	8	0	0	1	0	0	0	16
Total		496	1097	482	379	356	102	126	31	3069

g 21, 1 = 7

[ ] indicates the G matrix

i.e. the only changes taking place are those due to retirement. Under program expansion the elements of the D matrix will tend to be positive and under program contraction, negative.

The following pages give the G and L matrices used in these computations.

Projected Loss

Years Hence

		Number					Current	Total
GS Levels	3	5	10	3	5	10	Number	Percent
15	2.59	4.05	9.16	8	13	30	31	100
14	19.98	31.71	59.24	16	25	47	126	100
13	17.97	26.59	48.93	18	26	48	102	100
12	24.83	37.45	74.87	7	10	21	356	100
11	27.36	37.86	67.91	7	10	18	379	100
8-10	48.39	74.23	145.43	10	15	30	482	100
5-7	110.69	150.61	262.14	10	14	24	1097	100
1-4	75.54	96.69	140.44	15	20	28	496	100

$L_{8,1} = 75.54$

[ ] indicates the L matrix